

# A stochastic approach for modelling dynamic fracture of quasi-brittle materials

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## Abstract

The response of brittle and quasi-brittle materials is greatly influenced by their microstructural architecture. Models that assume material properties, particularly fracture strength, are spatially uniform fail to capture two important aspects of fracture in quasi-brittle materials. First, microscale inhomogeneities of quasi-brittle materials greatly influence eventual crack patterns by affecting where cracks are nucleated and how stress field is redistributed. Second, deterministic models cannot capture scatter in certain macroscopic measures, such as ultimate load and absorbed energy.

We use two approaches to model spatial and sample-to-sample variations of fracture strength. In the first, we use the Karhunen-Loeve method to realize random fields that are obtained by assumed first and second moments of the underlying fracture strength. We derive these moments by obtaining fracture strength of stochastic volume elements (SVEs) and using the moving-window method for a material with randomly distributed microcracks. In the second, we use the Weibull model to sample random values for fracture strength at the vertices of a discrete mesh. We compare fracture patterns obtained with these models for various input parameters, such as first and second moments and correlation length scale for the first approach and Weibull probability density function parameters for the second. We use initial and boundary conditions that correspond to a spatially uniform and temporally increasing stress field. This loading most clearly demonstrates the effect of material inhomogeneities on fracture patterns; unlike crack propagation from existing sources of stress concentration, e.g., crack tips, crack nucleation locations and, to some extent, crack-propagation directions are sensitive to the underlying realized random fields. We present results that demonstrate the importance of modeling the randomness of fracture strength, even for problems in which the stress field is initially highly nonuniform, *e.g.*, due to the presence of macroscopic cracks.